Toward a microscopic description of reactions involving exotic nuclei

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We propose an extension of the Continuum Discretized Coupled Channels (CDCC) method, where the projectile is described by a microscopic cluster model. This microscopic generalization (MCDCC) only relies on nucleon-target interactions, and therefore presents an important predictive power. Core excitations can be included without any further parameter. As an example we investigate the ^{7}Li+^{208}Pb elastic scattering at E_{lab} = 27 and 35 MeV. The ^{7}Li nucleus is known to present an α+t cluster structure, and is well described by the Resonating Group Method. An excellent agreement is obtained for the ^{7}Li+^{208}Pb cross sections, provided that breakup channels are properly included. We also present an application to inelastic scattering, and discuss future applications of the MCDCC.

The study of exotic nuclei is of major interest in current nuclear physics research [1, 2]. These nuclei present unusual properties, such as a low breakup threshold and an anomalously large rms radius. Experimentally they are investigated through secondary reactions induced by radioactive beams [3]. The first breakthrough in this field was the discovery of a large radius of the ^{11}Li isotope [4], and lead to the definition and introduction of "halo" nuclei in the nuclear nomenclature. A halo nucleus is considered as a tightly bound core nucleus surrounded by one or two weakly bound nucleons. Thanks to the recent development of experimental facilities, other exotic nuclei, such as ^{6}He, ^{8}B and ^{14}Be, can now be produced with high intensities. In recent years, the effects of low breakup threshold energies have been experimentally studied in various processes, such as elastic scattering [5], elastic breakup [6], and fusion [7]. As a general statement, the large rms radius of exotic nuclei has a strong impact on the nucleus-nucleus interaction, as it extends further the range of the nuclear component.

An accurate description of the breakup processes requires high quality reaction models. A scattering model essentially relies on two ingredients: (*i*) a description of the quantum scattering process itself; (*ii*) a reliable wave function that faithfully describes the exotic projectile. The detailed description of the projectile is a crucial issue in the field, as standard approximations, neglecting the structure of the colliding nuclei, are not appropriate.

At high energies, the Glauber model [8], using the eikonal approximation [9, 10] provides an accurate description of various cross sections. The early calculations, based on the adiabatic approximation, were recently extended to include excited or breakup states of the projectile [11, 12]. The eikonal approximation provides a rather great simplification of the Schrödinger equation. This makes it possible to perform two-body and three-body breakup calculations, with correct treatment of scattering boundary conditions [13, 14].

At low energies (i.e. typically around the Coulomb barrier) the eikonal approximation is not valid. In this energy regime, the Continuum Discretized Coupled Channel (CDCC) method has proved to be an accurate theoretical tool [15, 16]. It was originally developed for deuteron-induced reactions [17]. Owing to the low binding energy of the deuteron, the theoretical description of the d + nucleus elastic cross section could be significantly improved by including the coupling to the breakup channel (d $\rightarrow$ p+n). Accordingly, owing to to the low binding energies involved, the CDCC is an ideal reaction theory for exotic nuclei.

In standard CDCC calculations, the projectile is described by a two-body structure, where the constituents interact through an appropriate potential (fitting, for example, the ground-state energy). The internal Hamiltonian is then solved over a basis, and the associated eigenstates are used as a basis for the full projectile-target problem. Position-energy states are referred to as pseudostates (PS) and they provide an approximation of the core-fragment continuum. In addition to the textbook example d+^{58}Ni reaction [18], many other reactions have been recently investigated within this framework (see Ref.[19] for a recent review). The formalism has been extended further to three-body projectiles [20, 21] to deal with two-neutron halo nuclei such as ^{6}He and ^{11}Li, so-called Borromean nuclei.

These traditional CDCC calculations, however, present several shortcomings. The Hamiltonian associated with the system requires optical potentials between the target and the projectile constituents. If optical potentials are in general available for nucleons and α particles, rough approximations should often be used for beam nuclei such as ^{9}Li and ^{10}Be. Another limitation comes about from the potential model description of the projectile. If this approximation is, in most cases, reasonable, it may introduce inaccuracies in the cross section. In particular, core excitations are known to be important on many exotic nuclei, and the effect is absent from most CDCC calculations.

In this Letter, we propose a new approach to CDCC calculations, by using a microscopic cluster description of the projectile. In the microscopic CDCC approach (MCDCC), the projectile (with A_p nucleons) is described by a many-body Hamiltonian

$$H_0 = \sum_{i=1}^{A_p} t_i + \sum_{i<j=1}^{A_p} v_{ij}, \qquad (1)$$

where t_i is the kinetic energy operator of nucleon i, and v_{ij} a nucleon-nucleon interaction. Hamiltonian (1) is common to all microscopic theories such as the Fermionic Molecular Dynamics [22], the No-Core Shell Model [23], or the Variational Monte-Carlo method [24]. However, a fundamental issue in CDCC calculations is the ability of the model to describe continuum states of the projectile and how they influence the reaction dynamics. We therefore use here the cluster approximation, known as the Resonator Group Method (RGM) [25, 26], where an eigenstate of the Hamiltonian (1) is written as an anti-symmetric product of cluster wave functions. The RGM, and the equivalent Generator Coordinate Method (GCM, [27]), have been applied to spectroscopic and scattering properties of many systems (see Ref. [26] and references therein). In the present exploratory work, we consider ^{7}Li as projectile. The RGM-GCM is well known to reproduce many spectroscopic features of this weakly bound nucleus (as well as its mirror partner, ^{7}Be), by assuming an α + t structure (or α+^{3}He for the mirror partner) [28]. In other words, the ^{7}Li wave functions associated with H_0 are defined as,

$$\phi_k^{\ell jm} = \mathcal{A}\big[[\phi_\alpha \otimes \phi_t]^{1/2} \otimes Y_\ell(\Omega_\rho)\big]^{jm} g_k^{\ell j}(\rho), \qquad (2)$$

where ϕ_α and ϕ_t are shell model (SM) wave functions of the α and t clusters, l is the angular momentum and j, the total spin. In Eq. (2), ρ is the relative coordinate (see Fig. 1), and $\mathcal{A}$ is the 7-body anti-symmetrization operator which takes into account the Pauli exclusion principle among the 7 nucleons of the projectile. The relative wave function $g_k^{\ell j}(\rho)$, where k labels the bound and continuum states, are determined from the Schrödinger equation associated with H_0. In general the RGM equation is non-local [25]. The GCM is exactly equivalent to the RGM, but is better adopted to numerical calculations, as it makes use of Slater determinants (SD). The wave function (2) is therefore written as

$$\phi_k^{\ell jm} = \int f_k^{\ell j}(S)\Phi^{\ell jm}(S)\, dS, \qquad (3)$$

where S is the generator coordinate, $f_k^{\ell j}(S)$ the generator function, and $\Phi^{\ell jm}(S)$ a 7×7 projected Slater determinant with four $0s$ orbitals centered at $3R/7$, and three $0s$ orbitals centered at $4R/7$. Using the SD in the calculation of matrix elements of H_0 (and of other operators, such as the electromagnetic ones), is quite systematic, and can be extended to the p and sd shells, even with core excitations [29].

FIG. 1. Schematic picture of the projectile-target system, with a microscopic cluster structure of the projectile. Coordinates R and ρ are defined in the text.

Starting from the GCM functions for the projectile, the total Hamiltonian of the scattering system is,

$$H = H_0 + T_R + \sum_{i=1}^{A} V_{ti}(\boldsymbol{r}_i - \boldsymbol{R}), \qquad (4)$$

where $\boldsymbol{R}$ is the projectile-target relative coordinate (see Fig. 1) and V_{ti} are the nucleon-target interactions. The MCDCC approach presents many advantages: (1) the projectile wave functions are fully anti-symmetric, and not limited to bound states; (2) core excitations can be included in a straightforward way; (3) the model only relies on nucleon-target optical potentials. This potential is in general well known, and is independent of the projectile. The predictive power of the model is therefore expected.

The last step of the MCDCC is to project the total wave function onto one with specified spin, J, and parity, π,

$$\Psi^{JM\pi} = \frac{1}{R}\sum_{cL}\big[\phi_k^{\ell j} \otimes Y_L(\Omega_R)\big]^{JM} u_{cL}^{J\pi}(R), \qquad (5)$$

where L is the relative angular momentum, and the index c, stands for $c = (\ell, j, k)$. The radial wave functions $u_{cL}^{J\pi}(R)$ are obtained from the coupled-channels system of equations,

$$-\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dR^2} - \frac{L(L+1)}{R^2}\right]u_{cL}^{J\pi} + \sum_{c'L'} V_{cL,c'L'}^{J\pi} u_{c'L'}^{J\pi} = (E - E_c)u_{cL}^{J\pi}, \qquad (6)$$

where E_c are the projectile energies, and where the coupling potentials $V_{cL,c'L'}^{J\pi}(R)$ re obtained from the matrix elements,

$$V_{cc'}(\boldsymbol{R}) = \langle \Phi_k^{\ell j}| \sum_{i=1}^{A} V_{ti}(\boldsymbol{r}_i - \boldsymbol{R})|\Phi_{k'}^{\ell' j'}\rangle, \qquad (7)$$

and from additional algebraic coefficients. Equation (7) involves one-body matrix elements, which can be computed by using standard formula [30]. The system (6) is then solved by using the R-matrix method [31, 32]. The solution provides the scattering matrix for all $(J\pi)$ values, and consequently the various cross sections (elastic scattering, breakup, fusion, etc.).

As mentioned above, our first application of the MCDCC deals with ^{7}Li elastic scattering on a heavy target, which we take here to be ^{208}Pb. As the MCDCC involves heavy numerical calculations, we illustrate the power of the method in a simple case, where ^{7}Li is described as a α + t cluster structure. The system only involves $0s$ orbitals (with an oscillator parameter $b = 1.45$ fm) and core excitations are absent.

The ^{7}Li wave functions are defined from a discretization of Eq.(3) with 20 values of the generator coordinator S, ranging from 0.8 fm to 16 fm in steps of 0.8 fm. The nucleon-nucleon interaction v_{ij} (see Eq.(1)) is taken as the Minnesota force [28], complemented with a zero-range spin-orbit term [33]. Taking for the admixture parameter $u = 1.011$, and for the spin-orbit amplitude $S_0 = 20$ MeV.fm^5, one reproduces the $3/2^-$ ground state and the $1/2^-$ state energies simultaneously. In the α + t wave functions, $j_{max} = 7/2$ (with both parities), and pseudostates up to 20 MeV are included. Various tests have been performed to test the stability of the calculated cross sections against the cut-off energy. This microscopic cluster model is very similar to those used in the past to describe the spectroscopy of ^{7}Li and the ^{3}H$(\alpha,\gamma)^7$Li cross sections [34]. A test is provided by the electromagnetic transition probabilities and by the quadrupole moment. For the $B(E1, 3/2^- \rightarrow 1/2^-)$ value, the GCM gives $7.5\, e^2$.fm^4, in good agreement with experiment $8.3 \pm 0.5\, e^2$.fm^4. The theoretical and experimental values of the ground-state quadrupole moment are $-37.0\, e$ mb and $-40.6 \pm 0.8\, e$ mb, respectively.

The ^{7}Li wave functions (including the pseudo states) are then used to determine the coupling potentials (7). With our conditions, the number of ^{7}Li states are 6,6,6,5 for $j = 1/2^+$ to $j = 7/2^+$), and 7,7,6,7 for $j = 1/2^-$) to $j = 7/2^-$). The neutron-^{208}Pb optical potential (at $E_n = E_{lab}/7$) is taken Ref. [35]. The proton-^{208}Pb cross section at $E_p = E_{lab}/7$ is identical to the Rutherford cross section [36], and the corresponding interaction only involves the Coulomb potential.

The coupled-channel equations (6) are then solved with the R-matrix method, as alluded to above. For high partial waves, the number of (cL) values can be quite large (up to). Owing to the long-range nature of the dipole Coulomb potentials, large channel radii must be used. Many numerical tests have been performed to check the stability of the results against the chosen channel radius and the number of basis functions. In Fig. 2, we present the elastic scattering cross sections at $E_{lab} = 27$ and 35 MeV. The calculations have been performed by increasing the number of ^{7}Li states. Obviously, the single-channel approach, limited to the ^{7}Li ground state is not able to reproduce the data. At $E_{lab} = 27$ MeV, a slight improvement is obtained by including the $1/2^-$ excited state. At both energies, however, an excellent agreement can only be achieved by including all breakup channels up to $j = 7/2$. A non-microscopic CDCC calculation [37] requires a renormalization of the $\alpha-^{208}$Pb and the t $+^{208}$Pb optical potentials by a factor 0.6. Therefore our approach presents a more powerful predictive procedure as, contrary to the former, it contains no adjustable parameters.

FIG. 2. ^{7}Li+^{208}Pb elastic scattering at $E_{lab} = 27$ MeV (a) and 35 MeV (b). Dotted lines represent the calculations without breakup channels, and the solid lines are the full calculations with increasing α-t angular momentum j_{max}. Experimental data are from Ref. [38].

Our model can be further tested through the calculation of the inelastic cross section, presented in Fig. 3. This cross section is much smaller than the elastic one, and is more sensitive to the details of the wave function. Notwithstanding that no fitting procedure has been applied, the agreement with the data is reasonably fair. Here again, the role of the breakup channels is not negligible. In particular the second excited state, $j = 7/2^-$ slightly reduces the cross section.

Thus exploratory work on the ^{7}Li + ^{208}PB elastic scattering shows that the MCDCC is a powerful tool for the description of low-energy reactions of weakly bound nuclei, where breakup coupling is important. It is certainly particularly suited to the scattering of exotic nuclei, which present even lower breakup thresholds, enhancing the effect of these channels. The model is only based on nucleon-target optical potentials, which are available over a wide range of masses and bombarding energies. Without any renormalization factors, we have shown that ^{7}Li + ^{208}Pb elastic and inelastic cross sections data can be fairly well reproduced provided that breakup channels are properly included in the calculation. The present approach opens many new perspectives

FIG. 3. Inelastic ^{7}Li+^{208}Pb→^{7}Li($1/2^-$)+^{208}Pb cross section at E_{lab} = 27 MeV. The data are taken from Ref. [39] (black circles) and [37] (open circles). The MCDCC curves for $j_{max} = 3/2$ and $j_{max} = 5/2$ are superimposed at the scale of the figure.

in nucleus-nucleus reaction calculations at low energies. We concentrated here on ^{7}Li, a well known α + t cluster nucleus. However, extending Eq. (2) to include core excitations is quite feasible. In fact, many microscopic cluster structure calculations have been performed with core excitations (see, e.g., Ref. [40] for ^{11}Be, and Ref. [41] for ^{17}C). Calculations for these exotic nuclei are much more involved, but the model itself is identical. Besides, the present model can be easily extended to three-cluster projectiles, such as the Borromean two-neutron halo nuclei, ^{6}He and ^{11}Li, where RGM wave functions are available [42, 43]. Finally, other processes such as breakup reactions and fusion reactions, both of great current interest, can be described by simple generalizations of the present work.

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